Let P be the point $(-6, 1, -3)$. Let Q be the point $(-3, -7, 5)$. Let R be the point $(-4, -3, 2)$.
Let $\vec{u}$ be the vector with initial point P and terminal point Q.
Let $\vec{w}$ be the vector with initial point P and terminal point R.
Let $\vec{t} = 3\vec{j} - 7\vec{k}$.

[a] Write $3\vec{w} - 2\vec{u}$ as a linear combination of $\vec{i}$, $\vec{j}$ and $\vec{k}$.

$$\vec{u} = \overrightarrow{PQ} = <-3-(-6), -7-1, 5-(-3)> = <3, -8, 8>$$
$$\vec{w} = \overrightarrow{PR} = <-4-(-6), -3-1, 2-(-3)> = <2, -4, 5>$$
$$3\vec{w} - 2\vec{u} = <6, -12, 15> - <6, -16, 16> = <0, 4, -1> = 4\vec{j} - \vec{k}$$

[b] Find a vector of magnitude 5 perpendicular to both $\vec{u}$ and $\vec{w}$. (Do **NOT** use decimal approximations.)

$$\vec{u} \times \vec{w} = <-40-(-32), -(15-16), -12-(-16)> = <-8, 1, 4>$$
SANITY CHECK: $(\vec{u} \times \vec{w}) \cdot \vec{u} = -24 - 8 + 32 = 0$ AND $(\vec{u} \times \vec{w}) \cdot \vec{w} = -16 - 4 + 20 = 0$
$$\frac{5}{\|\vec{u} \times \vec{w}\|}(\vec{u} \times \vec{w}) = \frac{5}{\sqrt{64+1+16}} <-8, 1, 4> = \frac{5}{9} <-8, 1, 4> = <-\frac{40}{9}, \frac{5}{9}, \frac{20}{9}>$$

[c] Find **symmetric** equations for the line which passes through P and is also perpendicular to the plane $4x - 7z = 9$.

The direction vector of the line must be parallel to the normal vector of the plane
Let direction vector = normal vector = $<4, 0, -7>$
$$\frac{x+6}{4} = \frac{z+3}{-7}, \quad y = 1$$

[d] Find the **general** equation of the plane which passes through P, Q and R.

$\vec{u}$ and $\vec{w}$ both lie in the plane, so the normal vector of the plane must be perpendicular to both $\vec{u}$ and $\vec{w}$
Let normal vector = $\vec{u} \times \vec{w} = <-8, 1, 4>$
$$-8(x+6) + 1(y-1) + 4(z+3) = 0$$
$$-8x + y + 4z - 48 - 1 + 12 = 0$$
$$-8x + y + 4z - 37 = 0$$

ADDITIONAL QUESTIONS ON OTHER SIDE ➡

[e] A force represented by the vector $4\vec{j} - 5\vec{k}$ moves an object from P to R. Find the work done.

$$<0, 4, -5> \cdot \overrightarrow{PR} = <0, 4, -5> \cdot <2, -4, 5> = 0 - 16 - 25 = -41$$

[f] Find the area of triangle PQR. (Do **NOT** use decimal approximations.)

$$\tfrac{1}{2}\left\|\overrightarrow{PQ} \times \overrightarrow{PR}\right\| = \tfrac{1}{2}\|\vec{u} \times \vec{w}\| = \tfrac{1}{2}(9) = \tfrac{9}{2}$$

[g] Write $4\vec{i} - 7\vec{k}$ as the sum of 2 vectors, one parallel to $\vec{w}$ and one perpendicular to $\vec{w}$. (Do **NOT** use decimal approximations.)

$$PROJ_{\vec{w}} <4, 0, -7> = \frac{<4, 0, -7> \cdot \vec{w}}{\vec{w} \cdot \vec{w}}\vec{w} = \frac{<4, 0, -7> \cdot <2, -4, 5>}{<2, -4, 5> \cdot <2, -4, 5>} <2, -4, 5>$$

$$= \frac{8 + 0 - 35}{4 + 16 + 25} <2, -4, 5> = \frac{-27}{45} <2, -4, 5> = -\frac{3}{5} <2, -4, 5> = <-\frac{6}{5}, \frac{12}{5}, -3>$$

$$<4, 0, -7> - <-\frac{6}{5}, \frac{12}{5}, -3> = <\frac{26}{5}, -\frac{12}{5}, -4>$$

$$<4, 0, -7> = <-\frac{6}{5}, \frac{12}{5}, -3> + <\frac{26}{5}, -\frac{12}{5}, -4>$$

[h] Find the volume of the parallelepiped with vectors $\vec{u}$, $\vec{w}$ and $\vec{t}$ as adjacent edges.

$$|(\vec{u} \times \vec{w}) \cdot \vec{t}| = |<-8, 1, 4> \cdot <0, 3, -7>| = |0 + 3 - 28| = |-25| = 25$$

[i] If the points P, Q and $(1, a, b)$ are collinear, find the values of a and b.

The points are collinear if and only if $\overrightarrow{PQ}$ and $\overrightarrow{PS}$ are parallel (where S is the point $(1, a, b)$)

$\overrightarrow{PQ} = k\overrightarrow{PS} \Rightarrow <3, -8, 8> = k <1 - (-6), a - 1, b - (-3)> \Rightarrow <3, -8, 8> = k <7, a - 1, b + 3>$

$3 = 7k$ $\qquad k = \frac{3}{7}$ $\qquad k = \frac{3}{7}$

$-8 = k(a - 1) \Rightarrow \ -8 = \frac{3}{7}(a - 1) \Rightarrow \ a = -\frac{53}{3}$

$8 = k(b + 3)$ $\qquad 8 = \frac{3}{7}(b + 3)$ $\qquad b = \frac{47}{3}$

[j] If $\|\vec{v}\| = 5$, and the angle between $\vec{w}$ and $\vec{v}$ is $\frac{2\pi}{3}$ radians, find the magnitude of $\vec{w} \times \vec{v}$. (Do **NOT** use decimal approximations.)

$$\|\vec{w} \times \vec{v}\| = \|\vec{w}\|\|\vec{v}\|\sin\tfrac{2\pi}{3} = \sqrt{45}(5)(\tfrac{\sqrt{3}}{2}) = \tfrac{15\sqrt{15}}{2}$$

[k] Find **parametric** equations for the line which passes through Q and is also parallel to the line $\dfrac{1 - y}{3} = -z = \dfrac{x + 6}{4}$.

The direction vector of the new line must be parallel to the direction vector of the given line

Let direction vector of new line $=$ direction vector of given line $= <4, -3, -1>$

$x = -3 + 4t$

$y = -7 - 3t$

$z = 5 - t$